The Math Citadel

Modelling Network Reliability More Simply Using Probability

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Content

Networking is Complex

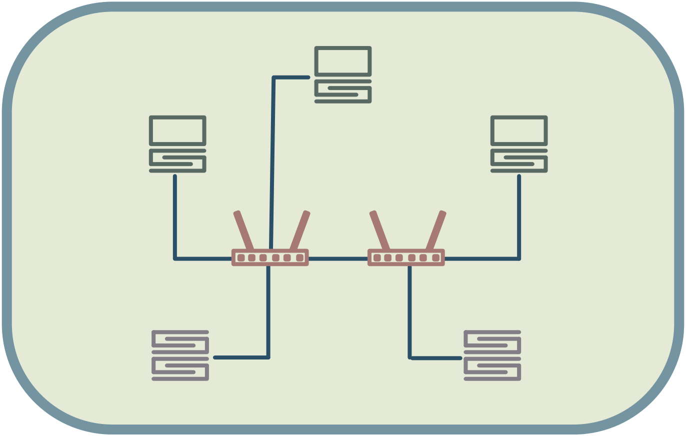
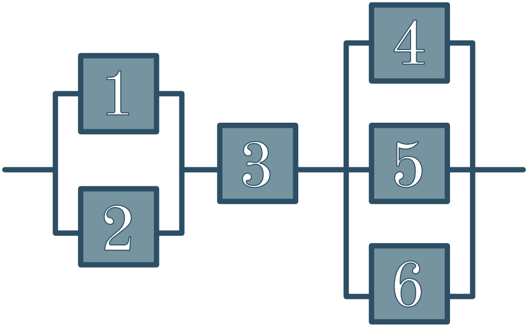


Figure: A sample physical network diagram

What's a reliability topology?
(aka reliability block diagram)

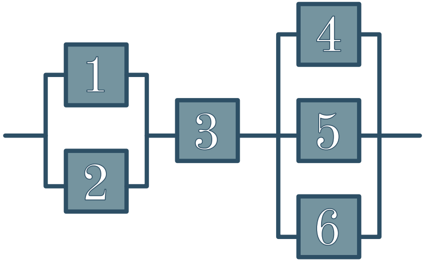


Path Sets

{1,3,4}
{1,3,5}
{1,3,6}
{2,3,4}
{2,3,5}
{2,3,6}

This tells us how the components in a network (or system) interact from a reliability standpoint.

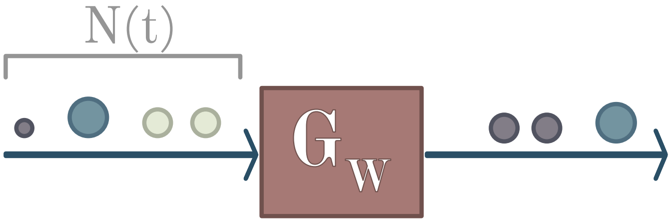
Q: What is the network failure probability?



Complications

- Traffic is random,
- Workload is random,
- Service times are random,
- Traffic may be correlated, and...
- Each component has a different survival function

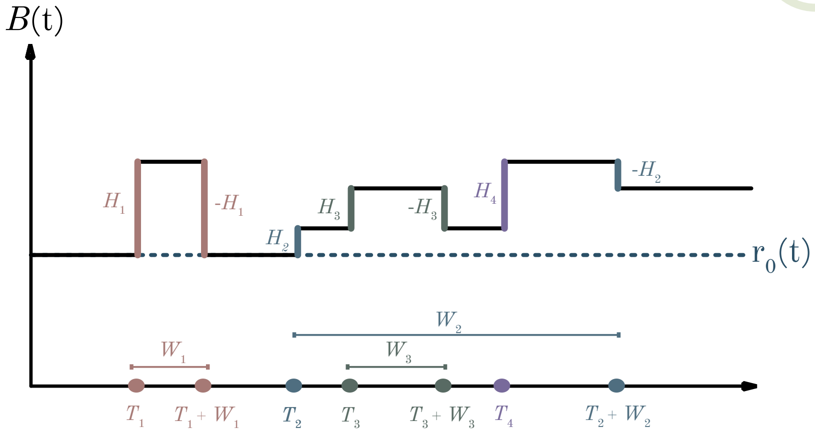
Focusing on a single device



Terms and notes

- $N(t)$- a counting process that counts the random number of arrivals to the device in $[0, t]$
- G_W- the probability distribution that governs the time to service any particular job
- Each job brings a workload (bubble size)
- The arrival times are random

Breakdown Rate Process (Hazard Rate)



$$\mathcal{B}(t) = r_0(t) + \sum_{j=1}^{N(t)} \mathcal{H}_j \mathbb{I}(T_j \leq t \leq T_j + W_j)$$

Breakdown Rate Process → Survival Function

Def: survival function

Let Y be the random time to device failure. The **survival function**, denoted $S_Y(t)$ is the cumulative probability that $Y > t$. That is

$$S_Y(t) := P(Y > t)$$

Breakdown Rate Process → Survival Function

Survival Function of the RSBR

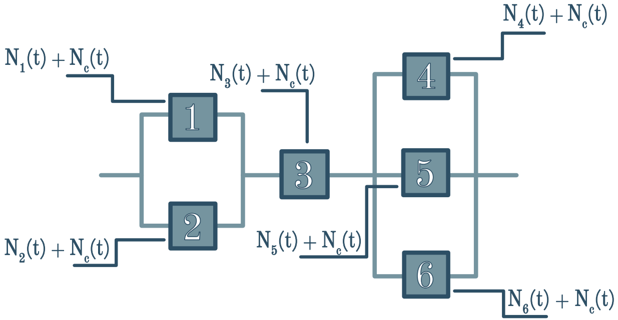
Suppose jobs arrive to a device according to a nonhomogenous Poisson process $\{N(t) : t \geq 0\}$ with intensity function $\lambda(t)$. Let $m(t) = E[N(t)] = \int_0^t \lambda(x)dx$. Suppose the arrival times $\{T_j\}_{j=1}^{N(t)}$ are independent, and the services times $\{W_j\}_{j=1}^{N(t)} \sim G_w$ be i.i.d. and mutually independent of service times. Assume the random stresses $H_j \sim H$ and are i.i.d. Then

$$\begin{aligned} S_Y(t) &= \exp\left(-\int_0^t \mathcal{B}(x)dx\right) \\ &= \exp\left(-\int_0^t r_0(x)dx - \mathrm{E}_{\mathcal{H}}\left[\mathcal{H}\int_0^t e^{-\mathcal{H}w}m(t-w)(1-G(w))dw\right]\right) \end{aligned}$$

It's closed form!

- fully interpretable
- extremely generalizable
- "all that remains" is parameter and distribution estimation
- We can build on this

Returning to Networks



Each device has its own...

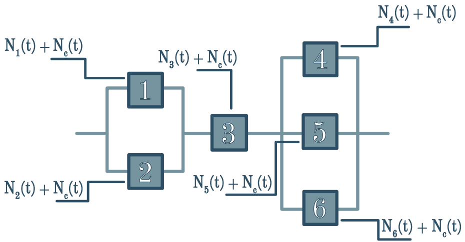
- arrival process
- service distribution
- job stresses distribution

...and the arrivals may be correlated ($N_c(t)$)

Def: structure function

Let x_i be the binary *state variable* for device i that indicates failure (0) or operational (1) status. Let $\mathsf{x} = (x_1, \dots x_n)$ be the *state vector* for the system. The **structure function** $\phi(\mathsf{x})$ is a function with binary outputs that indicates whether the entire system is running based on the input state vector.

Structure Functions and Survival Functions



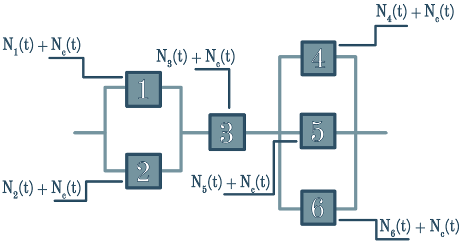
Structure Function for our Network

$$\phi(x) = [1 - (1 - x_1)(1 - x_2)]\, x_3\, [1 - (1 - x_4)(1 - x_5)(1 - x_6)]$$

Fun Fact

Every system is a parallel system of series subsystems, or a series system of parallel subsystems.

The Grand Finale

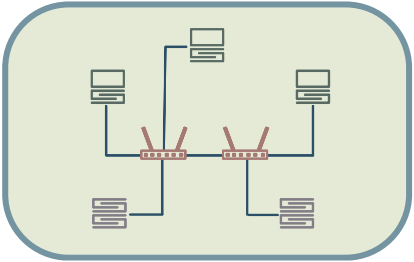


Substitute $S_i(t)$ in for each device i's state variable in the structure function.

For uncorrelated traffic, the survival function becomes

$$S_{Y_s}(t) = [1 - (1 - S_{Y_1})(1 - S_{Y_2})]\, S_{Y_3}\, [1 - (1 - S_{Y_4})(1 - S_{Y_5})(1 - S_{Y_6})]$$

What does this mean?



- We have an actual equation for any system's survival function
- The conditions are highly general
- Even these models are generalizable
- We're just left with the task of parameter estimation.

Dialogue: Your turn

Questions I have for you

- When I say stress, what do you think of?
- What layers of the network stack do you think this most applies to?
- Do you consider traffic flow in a network to behave more discretely (packets) or as a continuous flow? Does that change as you move up the stack?

What questions do you have for me? Comments? Inspirations?