Modelling Network Reliability More Simply Using Probability

Rachel Traylor, Ph.D.
Co-founder/Chief Scientist
www.themathcitadel.com
Content

Introduction

Single Device Reliability Model Under Random Workload

Building a Generalized Network Reliability Model

Conclusion
Networking is Complex

Figure: A sample physical network diagram
What’s a reliability topology?
(aka reliability block diagram)

This tells us how the components in a network (or system) interact from a reliability standpoint.
Q: What is the network failure probability?

Complications

- Traffic is random,
- Workload is random,
- Service times are random,
- Traffic may be correlated, and...
- Each component has a different survival function
Terms and notes

- $N(t)$ - a counting process that counts the random number of arrivals to the device in $[0, t]$
- $G_W$ - the probability distribution that governs the time to service any particular job
- Each job brings a workload (bubble size)
- The arrival times are random
Breakdown Rate Process (Hazard Rate)

\[ B(t) = r_0(t) + \sum_{j=1}^{N(t)} H_j \mathbb{1}(T_j \leq t \leq T_j + W_j) \]
Def: survival function

Let $Y$ be the random time to device failure. The survival function, denoted $S_Y(t)$ is the cumulative probability that $Y > t$. That is

$$S_Y(t) := P(Y > t)$$
Survival Function of the RSBR

Suppose jobs arrive to a device according to a nonhomogeneous Poisson process \( \{N(t) : t \geq 0\} \) with intensity function \( \lambda(t) \). Let \( m(t) = E[N(t)] = \int_0^t \lambda(x)dx \). Suppose the arrival times \( \{T_j\}_{j=1}^{N(t)} \) are independent, and the services times \( \{W_j\}_{j=1}^{N(t)} \sim G_w \) be i.i.d. and mutually independent of service times. Assume the random stresses \( H_j \sim H \) and are i.i.d. Then

\[
S_Y(t) = \exp \left( - \int_0^t B(x)dx \right) = \exp \left( - \int_0^t r_0(x)dx - E_H \left[ H \int_0^t e^{-Hw} m(t - w)(1 - G(w))dw \right] \right)
\]
Yikes! What good does that do me?

It’s closed form!

- fully interpretable
- extremely generalizable
- "all that remains" is parameter and distribution estimation
- We can build on this
Returning to Networks

Each device has its own...

▶ arrival process
▶ service distribution
▶ job stresses distribution

...and the arrivals may be correlated \((N_c(t))\)
Structure Functions and Survival Functions

**Def: structure function**

Let $x_i$ be the binary *state variable* for device $i$ that indicates failure (0) or operational (1) status. Let $x = (x_1, \ldots, x_n)$ be the *state vector* for the system. The *structure function* $\phi(x)$ is a function with binary outputs that indicates whether the entire system is running based on the input state vector.
Structure Functions and Survival Functions

Structure Function for our Network

\[
\phi(x) = [1 - (1 - x_1)(1 - x_2)] x_3 [1 - (1 - x_4)(1 - x_5)(1 - x_6)]
\]

Fun Fact

Every system is a parallel system of series subsystems, or a series system of parallel subsystems.
Substitute $S_i(t)$ in for each device $i$’s state variable in the structure function.

For uncorrelated traffic, the survival function becomes

$$S_{Y_s}(t) = [1 - (1 - S_{Y_1})(1 - S_{Y_2})] S_{Y_3} [1 - (1 - S_{Y_4})(1 - S_{Y_5})(1 - S_{Y_6})]$$
What does this mean?

- We have an actual equation for any system’s survival function
- The conditions are highly general
- Even these models are generalizable
- We’re just left with the task of parameter estimation.
Dialogue: Your turn

Questions I have for you

- When I say stress, what do you think of?
- What layers of the network stack do you think this most applies to?
- Do you consider traffic flow in a network to behave more discretely (packets) or as a continuous flow? Does that change as you move up the stack?
What questions do you have for me? Comments? Inspirations?